



## SOME BOUNDARY VALUE PROBLEMS SOLVED USING THE METHODS OF STOCHASTIC ANALYSIS

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### ABSTRACT:

In this paper, discuss some boundary value problem, can be solved using the method of stochastic analysis, perhaps the most celebrated example is Shizuo Kakutani's 1944 solution of the Dirichlet problems for the Laplace operator using Brownian motion. The idea of the stochastic method for solving this problem first, one finds a diffusion  $X$  whose infinitesimal generator  $A$  coincides with  $L$  on compactly supported  $C^2$  functions.

**Keywords:** - Boundary value problem, Stochastic analysis, Shizuo Kakutani's theorem, Laplace operator, generator supported function, Brownian motion.

### INTRODUCTION :

In mathematics, in the field of differential equations a boundary value problem is a differential equation together with a set of additional restraints called the boundary conditions. A solution to a boundary value problem is a solution to the differential equation which also satisfies the boundary conditions.

The boundary value problems in which the conditions are specified at more than one point. The crucial distinction between initial value problems and boundary value problems is that in the former case we are able to start an acceptable solution at its beginning and just march it along by numerical iteration to its end, while in the present case, the boundary condition at the starting point do not determine a unique solution to start with and a random choice among the solutions that satisfy these starting boundary conditions is almost certain not to satisfy the boundary condition at the other specified points.

In mathematics, the Neumann boundary condition is a type of boundary condition, named after Carl Neumann. When imposed on

an ordinary or a partial differential equation, it specifies the values that the derivative of a solution is to take on the boundary of the domain.

For an ordinary differential equation, for instance

$$Y'' + y = 0$$

the Neumann boundary conditions on the interval  $[a, b]$  take the form

$$y'(a) = \alpha \quad \text{and} \quad \alpha y'(b) = \beta \quad \text{where} \quad \alpha \quad \text{and} \quad \beta \quad \text{are} \quad \alpha$$

given numbers, In mathematics the Robin boundary condition is a type of boundary condition, named after Victor Gustave Robin (1855 – 1897). When imposed on an ordinary or a partial differential equation, it is a specification of a linear combination of the values of a function and the values of its derivative on the boundary of the domain, Robin should be pronounced as a French name, although many English-speaking mathematicians Anglicize the word.

Robin boundary conditions are a weighted combination of Dirichlet boundary conditions and Neumann boundary conditions. This contrasts to mixed boundary conditions, which

are boundary conditions of different types specified on different subsets of the boundary, Robin boundary conditions are also called impedance boundary conditions, from their application in electro magnetic problems.

Robin boundary conditions are commonly used in solving Sturm-Liouville problems which appear in many contexts in science and engineering used by Agmon (1).

The Born-von Karman boundary condition is important in solid state physics for analyzing many features of crystals, such as diffraction and the band gap. Modeling the potential of a crystal as a periodic function with the Born-von Karman boundary condition and plugging in Schrodinger's equation results in a proof of Bloch's theorem, which is particularly important in understanding the band structure of crystals used by Griffiths (3).

In mathematics a partial differential equation (P.D.E.) is a differential equation that contains unknown multivariable functions and their partial derivative. PDEs are used to formulate problems involving functions of several variables and are either solved by hand, or used to create a relevant computer model.

Partial differential equation (P.D.E.s) are equations that involve rates of change with respect to continuous variables. The configuration of a rigid body is specified by six numbers. Classic domains where PDEs are used include acoustics, fluid flow, electrodynamics, and heat transfer.

A partial differential equation (PDE) for the function  $u(x_1, x_2, \dots, x_n)$  is an equation of the form

$$F(x_1, x_2, \dots, x_n, u, \frac{\partial u}{\partial x_1}, \dots, \frac{\partial u}{\partial x_n}, \frac{\partial^2 u}{\partial x_1 \partial x_1}, \dots, \frac{\partial^2 u}{\partial x_1 \partial x_n}, \dots) = 0$$

If  $F$  is a linear function of  $u$  and its derivatives, then the PDE is called linear. Common examples of linear PDE include the heat equation, the wave equation, Laplace's equation, Helmholtz

equation, Klein - Gordon equation, and Poisson's equation from Folland (2).

A relatively simple PDE is

$$\frac{\partial u(x,y)}{\partial x} = 0$$

Hence the general solution of this equation is

$$u(x,y) = f(y)$$

Where  $f$  is an arbitrary function of  $y$ .

The analogous ordinary differential equation is

$$\frac{\partial u(x)}{\partial x} = 0$$

Which has the solution

$$u(x)=c \text{ where } c \text{ is any constant}$$

### OBSERVATION AND DISCUSSION :

A solution of a PDE is generally not unique, additional conditions must generally be specified on the boundary of the region where the solution is defined for instance, in the simple example above, the function  $f(y)$  can be determined if  $u$  is specified on the line  $x = 0$ .

### EXISTENCE AND UNIQUENESS :

Although the issue of existence and uniqueness of solutions of ordinary differential equation has a very satisfactory answer with the Picard-Lindelof theorem, that is far from the case for partial differential equations. The Cauchy-Kowalevski theorem states that the Cauchy problem for any partial differential equation, whose coefficient is analytic in the unknown function and its derivatives, has a locally unique analytic solution. Although this result might appear to settle the existence and uniqueness of solutions, there are examples of linear partial differential equations whose coefficient have derivatives of all orders but which have no solutions at all see Lewy (1957). Even if the solution of a partial differential equation exists and is unique, it may nevertheless have undesirable properties. The mathematical study of these questions is usually in the more powerful context of weak solutions.

An example of pathological behavior is the sequence of Cauchy problems for the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

With boundary conditions

$$u(x, 0) = 0$$

$$\frac{\partial u}{\partial y}(x, 0) = \frac{\sin(nx)}{n}$$

Where  $n$  is an integer

The derivative of  $u$  with respect to  $y$  approaches 0 uniformly in  $x$  as  $n$  increases, but the solution is

$$u(x, y) = \frac{\sin(ny) \sin(nx)}{n^2}$$

This solution approaches infinity if  $nx$  is not an integer multiple of  $\pi$  for any nonzero value of  $y$ . The Cauchy problem for the Laplace equation is called ill-posed or not well posed, since the solution does not depend continuously upon the data of the problem. Such ill-posed problems are not usually satisfactory for physical applications, by Taylor (5).

In PDEs, it is common to denote partial derivative using subscripts, That is :

$$u_x = \frac{\partial u}{\partial x}$$

$$u_{xy} = \frac{\partial^2 u}{\partial y \partial x} = \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial x} \right)$$

The equation for conduction of heat in two dimensions for a homogeneous body has

$$U_{tt} = c^2 u_{xx}, \text{ it's solution by method of separation of variable of the form } u(t, x) = T(t) \cdot X(x) \text{ Then } T'' + k^2 c^2 T = 0$$

$$X'' + k^2 X = 0$$

Where  $u(x, t)$  is temperature, and  $c$  is a positive constant that describes the rate of diffusion. The Cauchy problem for this equation consist in specifying

$$u(0, x) = f(x),$$

Where  $f(x)$  is an arbitrary function.

General solution of the heat equation can be found by the method of separation of variables, by John (4).

### CONCLUSION :

After analysis of boundary value problems observe that Cauchy boundary condition can be understood from the theory second order ordinary differential equation were to have a particular solution, one has to specify the value of the function and value of the derivative at a given initial boundary point.

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